EXERCISE 8

1. How many terms are there in each of the following expressions?

a) $2ab + c \div 3$
b) $6xy - 3y^2 - 2(x - y) + 2$
c) $6a \div 2 \times 3 \div (a + b)$
d) $\frac{3xy}{y} - 6(x + y - 3)$
e) $15 \div 3 - 4 \times 3 \div 2$
f) $a \times a \times a \times a + b \times b \times b$
g) $\frac{x-y}{x} - 4x(x + y + 3)$
h) $x^3 - 4\sqrt{x + y}$

2. Consider the expression $2x^3 - 4x^2 + 3x - 8$
a) What is the coefficient of x^2?
b) Write down the constant term.
c) What is the exponent of x in the first term?

3. Consider the expression $-3x^3 + 4ax^2 - 5a^2x + 6a^3 + 7$
a) What is the coefficient of x^3?
b) How many terms are there in the expression?
c) What is the exponent of a in the second term?
d) Write down the constant term.

4. Write algebraic expressions for the following:
a) Three times a number increased by 2.
b) Twice the product of two numbers increased by 3.
c) The product of two numbers decreased by 3.
d) The square of a number decreased by three times the cube root of the same number.
e) The sum of ten and a number divided by twice the number.

5. Simplify:

a) $13x^2 + 7 - 3x + 5x - 15x^2 - 7x$
b) $2a^2 - 3ab + b^2 + a^2 - ba + 7b^2$
c) $-6y^2 - 5y - 1 + 5 - y - y^2$
d) $7p^2 - 3pq + 4p^2 - 3q^2 + 6pq + 5q^2$
e) $-4u^2t + 3t^2u - 8 + 4u^2t + 6ut^2 + 6$

(Solutions p. 269 - 270)

6. If $x = -2$ and $y = 3$, determine the values of:

a) $(2x + y)(4x + 3y)$
b) $(-3x - 2y)(3x - y)$
c) $3(2x - y)$
d) $3x^3x + 7xy^2 + x$
e) $5x - 2y^2$
f) $\frac{3x+y^3}{y}$
g) $\frac{5xy}{x+y}$
h) $\sqrt{3x + 5y}$

7. Remove the brackets and simplify:

a) $-2a(3a - 5)$
b) $3x^2(-x + 2y - y^2)$
c) $m(-3m + 4)$
d) $-6p^2(-p - 3)$
e) $8x(-x^2 - 2x + 5)$
f) $(-2a)(5a^2) + 3a^3$
g) $(-4x)(2x^2) + 8x^3$
h) $4a(5a + 6) - 3(2a^2 + a)$
i) $ax(-3a + ax - 2x)$
j) $7p(p + 3) - 2p(p^2 + p - 5)$

8. Simplify:

a) $\frac{-18a^5b^3c^2}{9a^3b^3c}$
b) $\frac{25x^3y^2-5x^3y}{5x^2y}$
c) $\frac{10x^5y^3+15x^3y^2-5xy}{5xy}$
d) $\frac{12p^6q^4-9p^3q^2+6p^2q^2}{3p^2q}$
e) $\frac{7x^5y^3z^2-28x^3y^2z}{7x^3y^2z}$
f) $\frac{4c^3d^2+6c^2d^3-8c^5d}{-2cd}$
g) $\frac{20a^2b+35ab-15ab^2}{5ab}$
h) $\frac{8x^6y^3-4x^3y^6+4x^3y^3}{4x^3y^3}$
i) $\frac{4p^3q^3+8p^2q^2-6p^4q^2}{2p^2q^2}$
j) $\frac{8a^7+16a^5-32a^3}{8a^2}$

9. Simplify:
a) $5x - 8x$
b) $(4a)(-7a)$
c) $3ab \times 5b^2$
d) $4m + 2m \times 2 + 2 \times 2m$
e) $(-3p^3)^2$
f) $2xy^2 + 7xy - 5xy^2 + 10xy$
g) $2(p^2 - 2pq) - (p^2 - 5pq)$
h) $\frac{-28x^3y}{7x^2y}$

(Solutions p. 270 - 272)

ALGEBRAIC EQUATIONS

An equation is a mathematical sentence that is true for certain numbers. When we solve an equation, we find the value for the variable that makes the equation true.

1. SOLVING EQUATIONS BY INSPECTION

Sometimes it is easy to see the solution to an equation. In the equation $x + 2 = 10$ you can see the value of x is 8, because you know that $8 + 2 = 10$.
We say we solved the equation by inspection.

Example 1

Solve the following equations by inspection:

a) $x - 4 = 3$ b) $x + 5 = 8$
c) $3x = 12$ d) $\frac{x}{5} = 3$

Solution

a) $x - 4 = 3$
$\underline{x = 7}$ $\longleftarrow [3 + 4 = 7;\ therefore,\ x = 7]$

b) $x + 5 = 8$
$\underline{x = 3}$ $\longleftarrow [8 - 5 = 3;\ therefore,\ x = 3]$

c) $3x = 12$
$\underline{x = 4}$ $\longleftarrow [12 \div 3 = 4;\ therefore,\ x = 4]$

d) $\frac{x}{5} = 3$
$\underline{x = 15}$ $\longleftarrow [3 \times 5 = 15;\ therefore,\ x = 15]$

2. USE ADDITIVE AND MULTIPLICATION INVERSES TO SOLVE AN EQUATION

The additive inverse is the number added to a number to give a sum of 0 and **the multiplication inverse** is the number by which another number is multiplied to give a product of 1.

When solving an equation, we use the additive inverse to get all the constant terms on the right-hand side of the equation and all the variables on the left-hand side of the equation. We simplify both sides by adding like terms and then divide both sides by the coefficient of the variable.

Remember, whatever you do to one side of an equation, you must do to the other side. Now see how the following equation is solved.

$$4x - 7 = 5 + x$$

You want all the constants on the right-hand side. Add 7, the additive inverse of -7, to both sides of the equation. $\longrightarrow$ $4x - 7 + 7 = 5 + x + 7$

Add like terms: $-7 + 7 = 0$ *and* $5 + 7 = 12$ $\longrightarrow$ $4x = 12 + x$

You want all the variables on the left-hand side. Add $-x$, the additive inverse of x, to both sides of the equation. $\longrightarrow$ $4x - x = 12 + x - x$

Add like terms: $4x - x = 3x$ *and* $x - x = 0$ $\longrightarrow$ $3x = 12$

The multiplication inverse of 3 is ⅓. To multiply by ⅓, is the same as to divide by 3. Therefore, divide both sides of the equation by 3. $\longrightarrow$ $\frac{3x}{3} = \frac{12}{3}$

$$\underline{x = 4}$$

Example 2

a) Solve for x: $x + 4 = 29$ b) Solve for y: $-2y = 16$

Solution

a) $x + 4 = 29$

$x + 4 - 4 = 29 - 4 \longleftarrow$ [Add -4, the additive inverse of 4, to both sides of the equation]

$\underline{x = 25} \longleftarrow [4 - 4 = 0 \text{ and } 29 - 4 = 25]$

b) $-2y = 16$

$\frac{-2y}{-2} = \frac{16}{-2} \longleftarrow$ [Divide both sides by -2]

$\underline{y = -8} \longleftarrow [(-) \div (-) = + \text{ and } (+) \div (-) = -]$

Example 3

Solve the following equations:

a) $3x + 4 = 13 - 6x$ b) $-5a - 8 = 2 - 3a$

Solution

a) $3x + 4 = 13 - 6x$

$3x + 4 - 4 = 13 - 6x - 4 \longleftarrow$ [Add -4, the additive inverse of 4, to both sides]

$3x = 9 - 6x \longleftarrow$ [Simplify both sides $\rightarrow$ add like terms]

$3x + 6x = 9 - 6x + 6x \longleftarrow$ [Add $6x$, the additive inverse of $-6x$, to both sides]

$9x = 9 \longleftarrow$ [Simplify both sides, add like terms]

$\frac{9x}{9} = \frac{9}{9} \longleftarrow$ [Divide both sides by 9]

$\underline{x = 1}$

b) $-4a - 8 = 2 - 3a$

$-4a - 8 + 8 = 2 - 3a + 8 \longleftarrow$ [Add 8 to both sides]

$-4a = 10 - 3a$

$-4a + 3a = 10 - 3a + 3a \longleftarrow$ [Add $3a$ to both sides]

$-a = 10 \longleftarrow$ [Simplify both sides]

$\frac{-a}{-1} = \frac{10}{-1} \longleftarrow$ [Divide both sides by -1, $(-) \times (-) = +$ and $(+) \div (-) = -$]

$\underline{a = -10}$

> When brackets are used in an equation, first remove the brackets and then solve the equation.

Example 4

Solve the following equations:

a) $-3(6y + 4) = 6$ b) $3(x - 2) = 2(x + 3)$

Solutions

a) $-3(6y + 4) = 6$

$-18y - 12 = 6 \longleftarrow$ [Multiply each term inside bracket by -3]

$-18y - 12 + 12 = 6 + 12 \longleftarrow$ [Add 12 to both sides]

$-18y = 18 \longleftarrow$ [Simplify both sides; $-12 + 12 = 0$ and $6 + 12 = 18$]

$\frac{-18y}{-18} = \frac{18}{-18} \longleftarrow$ [Divide both sides by -18; Remember: $(+) \div (-) = -$]

$\underline{y = -1}$

b) $3(x - 2) = 2(x + 3)$

$3x - 6 = 2x + 6 \longleftarrow$ [Multiply each term inside bracket by number in front of bracket]

$3x - 6 - 2x + 6 = 2x + 6 - 2x + 6 \longleftarrow$ [Add $-2x$ and 6 simultaneously to both sides, it is shorter]

$\underline{x = 12} \longleftarrow$ [Simplify both sides. Note: $-6 + 6 = 0$ and $2x - 2x = 0$]

3. USE LAWS OF EXPONENTS TO SOLVE EQUATIONS

When the unknown variable is part of the exponent, we can solve the equation by getting the bases to be the same on both sides of the equation. If the bases are the same, the exponents are equal.

If $a^n = a^m$ then $n = m$

Consider the following example:

Example 5

Solve the following equations:

a) $3^x = 27$ b) $2^{a-1} = 8$

Solution

a) $3^x = 27$

$3^x = 3^3 \longleftarrow \left[\begin{matrix} \textit{Make the bases the same, write} \\ 27 \textit{ to base } 3 \rightarrow 27 = 3 \times 3 \times 3 = 3^3 \end{matrix}\right]$

$\underline{x = 3} \longleftarrow \left[\begin{matrix} \textit{The bases are the same,} \\ \textit{equate the exponents} \end{matrix}\right]$

b) $2^{a-1} = 8$

$2^{a-1} = 2^3 \longleftarrow \left[\begin{matrix} \textit{You want both sides with base } 2 \\ \textit{write } 8 \textit{ to base } 2 \rightarrow 8 = 2^3 \end{matrix}\right]$

$a - 1 = 3 \longleftarrow [\textit{Bases are equal, equate the exponents}]$

$a - 1 + 1 = 3 + 1 \longleftarrow [\textit{Add the additive inverse of } -1 \textit{ to both sides}]$

$\underline{a = 4}$

When using a square root to solve an equation, remember that your solution may have a negative or a positive answer.

If $x^2 = 9$, then $x = \pm\sqrt{9}$ therefore, $x = -3$ or $x = 3$

Example 6

Solve the following equations:

a) $2x^2 = 50$ b) $p^2 - 5 = 4$

Solution

a) $2x^2 = 50$

$x^2 = 25 \longleftarrow [\textit{First divide by 2, the coefficient of } x^2]$

$x = \pm\sqrt{25} \longleftarrow [x = \pm \textit{the square root of } 25]$

$\underline{x = 5 \text{ or } x = -5}$

b) $p^2 - 5 = 4$

$p^2 - 5 + 5 = 4 + 5 \longleftarrow [\textit{Add 5 to both sides of the equation}]$

$p^2 = 9$

$p = \pm\sqrt{9} \longleftarrow [\textit{Find square root on both sides}]$

$\underline{p = 3 \text{ or } p = -3}$

4. USE EQUATIONS TO SOLVE PROBLEMS

Algebraic equations can be used to solve problems. We set up an equation to describe the given information and then solve the equation. The unknown value is represented by a letter, usually x.

Example 7

A mother is three times as old as her daughter. Their combined ages are 52. How old are they respectively?

Solution

$Let\ the\ age\ of\ the\ daughter = x\ years$
$Age\ of\ the\ mother\quad = 3x\ years$ $[Mother\ is\ 3\ times\ as\ old,\ 3 \times x]$
$Age\ of\ mother + age\ of\ daughter = 52$
$Set\ up\ an\ equation\ to\ represent\ this\ information.$

$$\underbrace{Age\ of\ daughter}_{x} + \underbrace{age\ of\ mother}_{3x} = 52$$
$$x + 3x = 52$$
$$4x = 52 \quad \longleftarrow [3x + x = 4x]$$
$$x = 13 \quad \longleftarrow [Divide\ by\ 4]$$

Daughter is 13 years old and mother is 39 years old

Example 8

The sum of three consecutive numbers is 126. Find the numbers.

Solution

$Let\ the\ first\ number = x$
$The\ second\ number\quad = x + 1$
$The\ third\ number\quad = x + 2$
$The\ sum\ of\ the\ three\ numbers\ is\ 126.\ Now\ set\ up\ an\ equation.$

$$\underbrace{number\ 1}_{x} + \underbrace{number\ 2}_{x+1} + \underbrace{number\ 3}_{x+2} = 126$$
$$x + x + 1 + x + 2 = 126$$
$$3x + 3 = 126 \quad \longleftarrow [Add\ like\ terms]$$
$$3x + 3 - 3 = 126 - 3 \quad \longleftarrow [Add\ -3\ to\ both\ sides]$$
$$3x = 123 \quad \longleftarrow [Simplify\ both\ sides]$$
$$x = 41 \quad \longleftarrow [Divide\ both\ sides\ by\ 3]$$

The numbers are 41; 42 and 43

Example 9

Five times a number, increased by 8, equals 63. Find the number.

Solution

$Let\ the\ number = x;\ \ then\ 5\ times\ the\ number = 5x\ .$
$Now\ set\ up\ an\ equation.$

$$\underbrace{5 \times the\ number,}_{5x}\ \underbrace{increased\ by\ 8}_{8} = 63$$
$$5x + 8 = 63$$
$$5x + 8 - 8 = 63 - 8 \quad \longleftarrow [Add\ -8\ to\ both\ sides]$$
$$5x = 55 \quad \longleftarrow [Divide\ both\ sides\ by\ 5]$$
$$x = 11$$

The number is 11

Example 10

The length of a rectangle is 6 metres more than its breadth. Calculate the length of the sides of the rectangle if the perimeter is 44 metres.

Solution

$Let\ the\ breadth = x$ m and $the\ length = x + 6$ m
$The\ perimeter,\ 2l + 2b = 44.\ Now\ set\ up\ an\ equation.$

$$\underbrace{2(length)}_{2(x+6)} + \underbrace{2(breadth)}_{2(x)} = 44 \quad [perimeter\ of\ rectangle = 2l + 2b]$$
$$2(x + 6) + 2(x) = 44$$
$$2x + 12 + 2x = 44 \quad \longleftarrow [Remove\ the\ brackets]$$
$$4x + 12 - 12 = 44 - 12 \quad \longleftarrow \left[\begin{matrix} 2x + 2x = 4x; \\ add\ -12\ to\ both\ sides \end{matrix}\right]$$
$$4x = 32$$
$$x = 8 \quad \longleftarrow [Divide\ both\ sides\ by\ 4]$$

Die breadth = 8 metres and length = 8 + 6 = 14 metres

5. TABLES OF ORDERED NUMBER PAIRS

An ordered number pair consists of two numbers that are written in a specific order, e.g. (5; 2).
The first number represents the input value (x-value) and the second number represents the output value (y-value).

Example 11

Complete the table alongside containing the x- and y- values for the equation $y = 2x + 1$.

x	-3		5	
y		3		15

Solution

Column 1: $y = 2(-3) + 1$ ⟵[*Substitute* $x = -3$ *into the equation*]
$y = -6 + 1$
$y = -5$

Column 2: $3 = 2x + 1$ ⟵[*Substitute* $y = 3$ *into the equation*]
$3 - 1 = 2x + 1 - 1$ ⟵[*Add* -1 *to both sides*]
$2 = 2x$ ⟵[*Simplify both sides*]
$1 = x$ ⟵[*Divide both sides by* 2]

Column 3: $y = 2(5) + 1$ ⟵[*Substitute* $x = 5$ *into the equation*]
$y = 10 + 1$
$y = 11$

Column 4: $15 = 2x + 1$ ⟵[*Substitute* $y = 15$ *into the equation*]
$15 - 1 = 2x + 1 - 1$ ⟵[*Add* -1 *to both sides*]
$14 = 2x$ ⟵[*Simplify*]
$7 = x$ ⟵[*Divide both sides by* 2]

Now complete the table:

x	-3	**1**	5	**7**
y	$\mathbf{-5}$	3	**11**	15

Example 12

The table shows the x- and y-values for the equation $y = x^2 - 1$.
Complete the table.

x	-2	1		
y			8	15

Solution

Column 1: $y = (-2)^2 - 1$ ⟵[*Substitute* $x = -2$ *into the equation*]
$y = 4 - 1$
$y = 3$

Column 2: $y = (1)^2 - 1$ ⟵[*Substitute* $x = 1$ *into the equation*]
$y = 1 - 1$
$y = 0$

Column 3: $8 = x^2 - 1$ ⟵[*Substitute* $y = 8$ *into the equation*]
$8 + 1 = x^2 - 1 + 1$ ⟵[*Add* 1 *to both sides*]
$9 = x^2$ ⟵[*Simplify*]
$x = \pm 3$ ⟵[$x = \pm\sqrt{9} = \pm 3$]

Column 4: $15 = x^2 - 1$ ⟵[*Substitute* $y = 15$ *into the equation*]
$15 + 1 = x^2 - 1 + 1$ ⟵[*Add* 1 *to both sides*]
$16 = x^2$ ⟵[*Simplify both sides*]
$\pm 4 = x$ ⟵[$x = \pm\sqrt{16} = \pm 4$]

Now complete the table:

x	-2	1	$\mathbf{\pm 3}$	$\mathbf{\pm 4}$
y	**3**	**0**	8	15

<u>**EXERCISE 9**</u>

1. Solve the following equations by inspection:

a) $x + 3 = 10$	b) $x - 4 = 8$
c) $5a = 15$	d) $p - 5 = 10$
e) $2x = 14$	f) $x + 2 = 12$
g) $3a = 27$	h) $16 - x = 10$
i) $4p = 16$	j) $14 - a = 7$

2. Solve the following equations:

a) $x + 5 = 22$	b) $x + 3 = 17$
c) $2x - 4 = 18$	d) $5p = 18 + 2p$
e) $5x - 1 = 9$	f) $3a - 5 = 2a + 7$
g) $2p + 2 = 23 + p$	h) $18 - a = 26 - 3a$
i) $-3b - 8 = 7 + 2b$	j) $-2x - 5 = -17$

3. Solve for x:

a) $5(x + 1) = 8 - (x - 3)$	b) $2(2x - 3) = -2(1 - x)$
c) $3(x - 3) = 2(x - 4)$	d) $3x = -31 - 5(2x - 1)$
e) $7(x - 3) - 7 = -2(3x + 1)$	

4. Five times a certain number increased by 12 equals 67. Determine the number.

5. The sum of three consecutive numbers equals 45. Determine the numbers.

6. A book cost three times as much as a pen. Together they cost R84. What does a pen cost?

7. Jarine is five years younger than her brother. In three years' time the sum of their ages will be 29. How old is Jarine at the moment?

(Solutions p. 273 - 274)

8. The length of a rectangle is twice its breadth. If the perimeter of the rectangle is 72 cm, determine the length of the rectangle.

9. A taxi charges a fixed tariff of R20 plus R2,50 per kilometre. If Jessica pays R35, how far has she travelled?

10. Four times a certain number, increased by 15 equals 47. Determine the number.

11. Hein is eight years older than Liam. Six years ago Hein was three times as old as Liam. How old is Liam at the moment?

12. Divide a piece of garden hose of 16 m into two sections so that the one piece is 550 mm longer than twice the other piece. Determine the length of the two pieces of hose.

13. Wayne has 22 sweets and Niel has 18 sweets. How many sweets should Niel give Wayne so that Wayne will have four times more sweets than he does?

14. A motorist completes his journey in five hours at an average speed of 120 km/h. Calculate the distance he has travelled if he has stopped for 30 minutes to fill the tank.

(Solutions p. 275)

GRAPHS

1. ANALISE AND INTERPRET GRAPHS

Graphs can be used to represent the relationship between two variables.

> The **dependent variable** is the variable which is being observed and is affected by the other variable.
>
> The **independent variable** is not affected by the other variable, it is the variable being manipulated.

The following trends and features of graphs are very important when we analyse and interpret graphs.

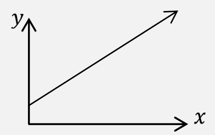
Features of graphs

In Grade 7 you have been introduced to increasing, decreasing and constant graphs as well as linear and non-linear graphs.

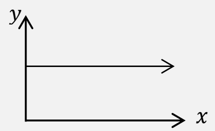
Increasing, constant or decreasing graphs

A graph shows how the dependent variable changes when the independent variable increases.

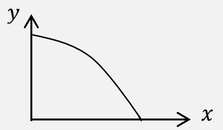
If the y-values increase as the x-values increase, the graph is **increasing.**



If the y-values stay constant as the x-values increase, the graph is **constant.**

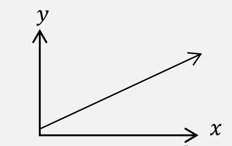


If the y-values decrease as the x-values increase the graph is **decreasing.**

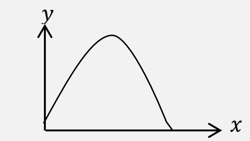


Linear and non-linear graphs

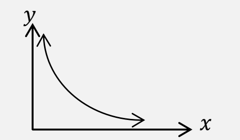
If a quantity increases or decreases at a constant speed, it is called a **linear change** and the graph forms a straight line.
If the change is not constant, it is called a **non-linear change** and the graph forms a curve.



Linear → straight line



Non-linear → curve



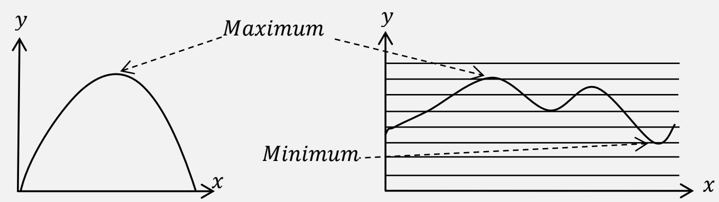
Non-linear → curve

We will now study maximum and minimum values of graphs as well as discrete and continuous data.

Maximum and minimum values

When a ball is thrown into the air, it will travel upwards to a certain point and then come down as it falls to the ground.
The temperature increases during the day and it decreases at night.
The weather report always indicates the maximum and minimum temperature of the biggest cities and towns.

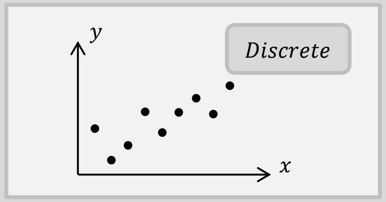
A maximum value is the largest value on the graph.
A minimum value is the smallest value on the graph.

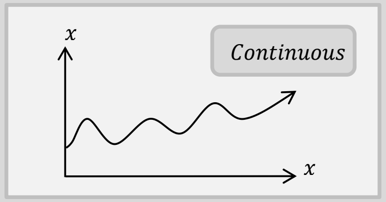


Discrete and **continuous** graphs

Discrete data is data that is counted. It can take only particular values and is represented on a graph as a set of plotted points not connected by a line.

Continuous data is data that is measured. It is not restricted to defined separate values but may take any value. It is represented on a graph as a curve or solid straight line.

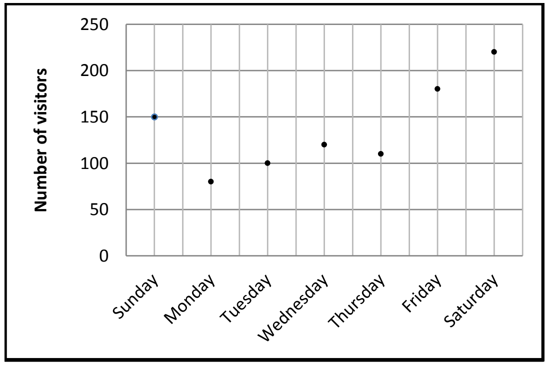




Example 1

The graph shows the number of tourists who have visited a game farm during a week.

a) What is the dependent variable?
b) Is this data discrete or continuous?
c) What is the maximum number of visitors?
d) On which day did the least number of tourists visit the game farm?



Solution

a) The number of visitors. ⟵ [*It is the variable being observed*]

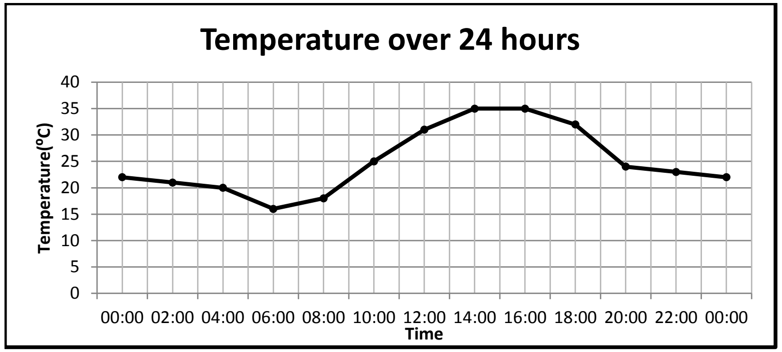
b) Discrete data. ⟵ [*The visitors are counted; there cannot be* 110,5 *visitors*]

c) The maximum number of visitors is 220.

d) On Monday the least number of tourists visited the farm.

Example 2

The graph shows the temperature of a town measured over 24 hours.



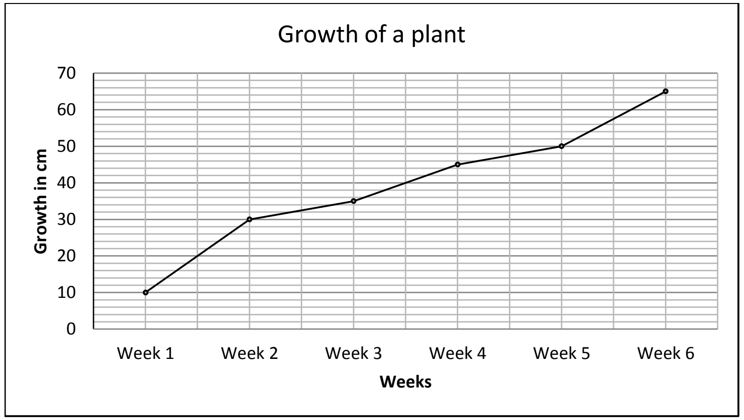
a) What is the maximum temperature?
b) Is the data discrete or continuous?
c) During which period is the temperature constant?
d) During which period does the temperature increase?

Solution

a) The maximum temperature is 35 °C.
b) The data is continuous.
c) The temperature is constant from 14:00 until 16:00.
d) The temperature increases from 06:00 until 14:00.

Example 3

The graph shows the growth of a plant measured for six consecutive weeks.



a) Does the graph represent discrete or continuous data?
b) What is the growth during the second week?
c) During which week was the growth the greatest?
d) Is the graph increasing or decreasing?

Solution

a) It represents continuous data because the plant keeps on growing.
b) The growth during the second week was 5 cm.
c) During the first week (the growth of the plant was 20 cm).
d) The graph is increasing.

2. DRAW GRAPHS

2.1 Draw global graphs

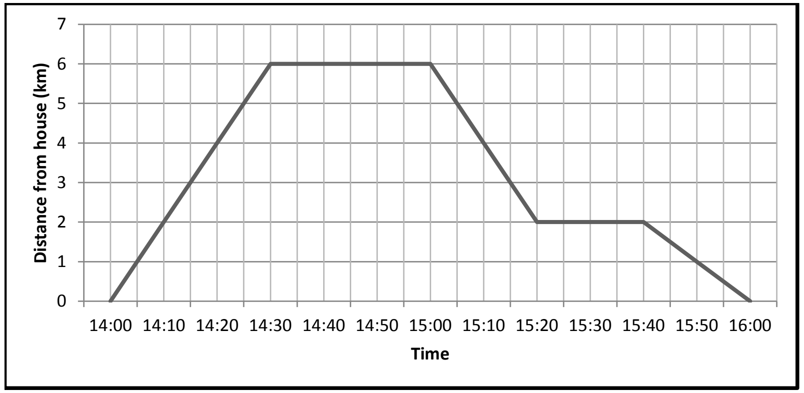
Example 4

James leaves his home at 14:00 and travels with his bicycle at a constant speed to a shopping mall six kilometres away. He arrives at the mall at 14:30 and shops for 30 minutes. Then he returns home. On his way home, he stops at a friend's house and they spend 20 minutes together. He arrives home at 16:00.

a) Draw a distance-time graph to represent James's trip.
b) During which time intervals is the graph constant?

Solution

a) *Time is the independent variable. Indicate time on the horizontal axis and the distance from the house on the vertical axis.*



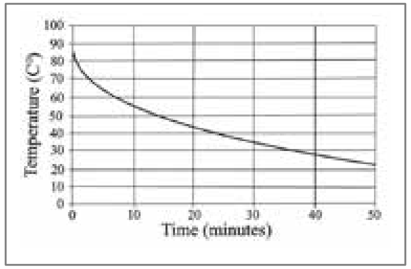
b) From 14:30 until 15:00 and again from 15:20 until 15:40.

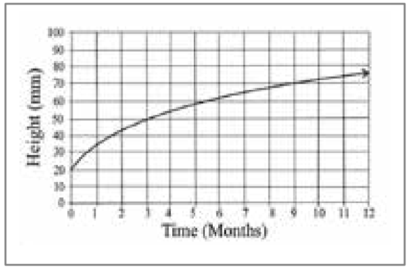
Example 5

Draw a graph to represent each of the following situations.

a) The temperature of a hot drink cooling down.
b) The height of a plant over a period of 12 months.

Solution

a) 

b) 

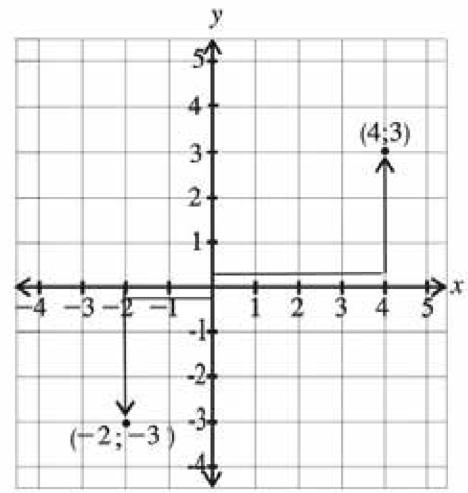
2.2 Draw graphs of ordered pairs

An ordered number pair is a pair of numbers written in a specific order. The numbers are written in brackets and is separated by a semicolon(;). The first number represents the x-value and the second number the y-value. We can plot ordered number pairs on the Cartesian plane.

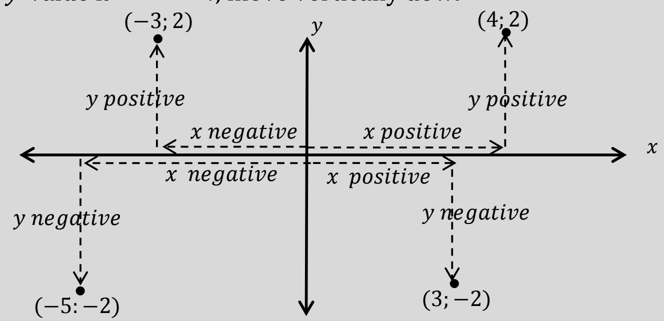
The Cartesian plane consists of two perpendicular number lines. The horizontal line is called the x-axis and the vertical line is called the y-axis. The point of intersection of the two axes is called the origin. The origin is the point $(0; 0)$, the x-coordinate is 0 and the y-coordinate is 0.

The point $(4; 3)$ is 4 units to the right of the origin and 3 units up.
To plot the point, from the origin, move 4 units to the right on the x-axis and then move 3 units up.

To plot the point $(-2; -3)$, from the origin, move 2 units to the left on the x- axis and then 3 units down.



If the x-value is positive, move horizontally to the right on the x-axis.
If the x-value is negative, move horizontally to the left on the x-axis.
If the y-value is positive, move vertically up.
If the y-value is negative, move vertically down



Example 6

Plot the following ordered pairs on the Cartesian plane.

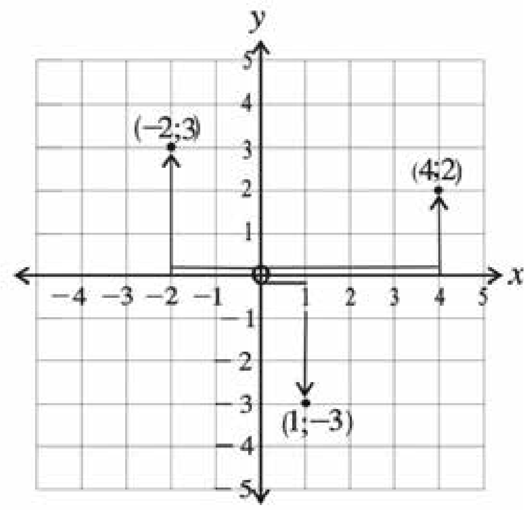
a) $(-2; 3)$ b) $(1; -3)$ c) $(4; 2)$

Solution

a) $(-2;3)$
$2\ units\ horizontally\ to\ the\ left,\ 3\ units\ vertically\ up$

b) $(1;\ -3)$
$1\ unit\ horizontally\ to\ the\ right,\ 3\ units\ vertically\ down$

c) $(4;2)$
$4\ units\ horizontally\ to\ the\ right,\ 2\ units\ vertically\ up$



To represent the relationship between two variables graphically, we can plot the ordered pairs on the Cartesian plane and join the points to form a graph.

Example 7

a) Complete the following table for the equation $y = 2x + 1$.

x	-3	-2	-1	0	1	2
y						

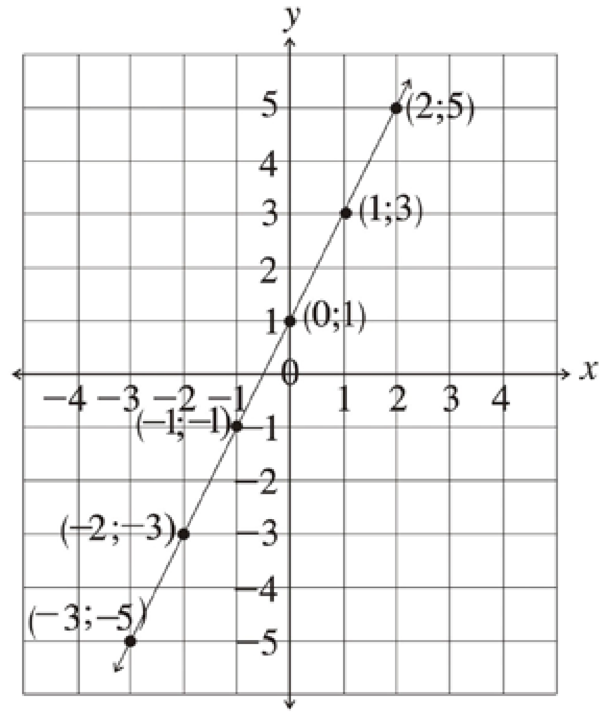
b) Plot the points on the Cartesian plane and join the points to form a graph.

Solution

a) For $x = -3$: $y = 2(-3) + 1 = -6 + 1 = -5$
For $x = -2$: $y = 2(-2) + 1 = -4 + 1 = -3$
For $x = -1$: $y = 2(-1) + 1 = -2 + 1 = -1$
For $x = 0$: $y = 2(0) + 1 = 0 + 1 = 1$
For $x = 1$: $y = 2(1) + 1 = 2 + 1 = 3$
For $x = 2$: $y = 2(2) + 1 = 4 + 1 = 5$

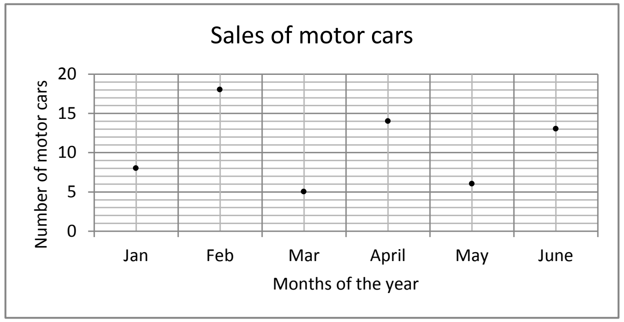
x	-3	-2	-1	0	1	2
y	-5	-3	-1	1	3	5

b) Plot the points as in example 6 and join the points.



EXERCISE 10

1. The graph shows the number of motor cars sold by a salesman the first six months of the year.



a) Does the graph represent discrete or continuous data?
b) During which month were the most cars sold?
c) What was the maximum sales?
d) What was the minimum sales?

2. Draw sketch graphs to represent the following:
a) The number of cool drinks you buy and the amount you pay.
b) The temperature in a certain town over a period of 24 hours.
c) The height of a mealie plant over a period of 10 weeks.

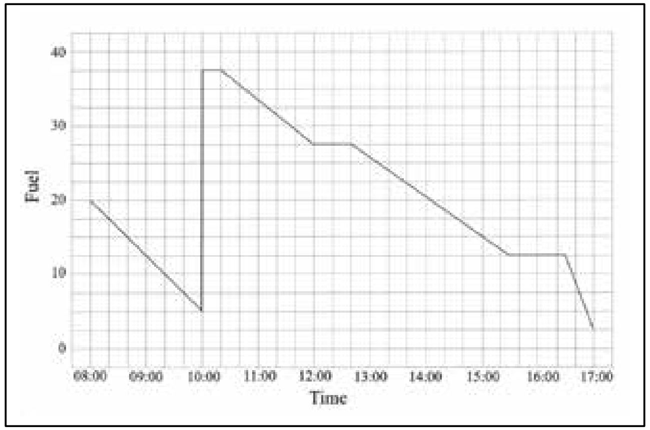
3. A farmer leaves his farm at 08:00 and travels into town 100 km away. He stops in town for a short while and then returns to the farm. The table shows the distance he has covered every hour.

Time	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00
Distance (km)	0	20	75	100	100	80	40	25	0

a) Draw a distance-time-graph to represent his journey.
b) When did he arrive in town?
c) For how long did he stop in town?
d) During which hour did he travel the fastest?

(Solutions p. 276)

4. A motorist left Bloemfontein at 08:00 and reached George at 17:00. He stopped for petrol and for tea, for lunch and again for coffee. The graph represents the number of litres of petrol in the tank of the car at the end of each hour.



a) How many litres were in the tank at the start of his journey?
b) When did he stop to buy petrol and to have tea?
c) How many litres of petrol did he buy?
d) How long did he stop for lunch?
e) Why is the graph descending to the right?

5. Plot the following points on the Cartesian plane:

a) $(2;\ -3)$ b) $(4; 1)$ c) $(-3; -1)$ d) $(-1: 3)$

6. Complete the following tables for the given equations and plot the points on the Cartesian plane. Join the points to form a graph

a) $y = 2x - 1$

x	-2	-1	0	1	2
y					

b) $y = x + 2$

x	-2	-1	0	1	2
y					

c) $y = 3x$

x	-2	-1	0	1	2
y					

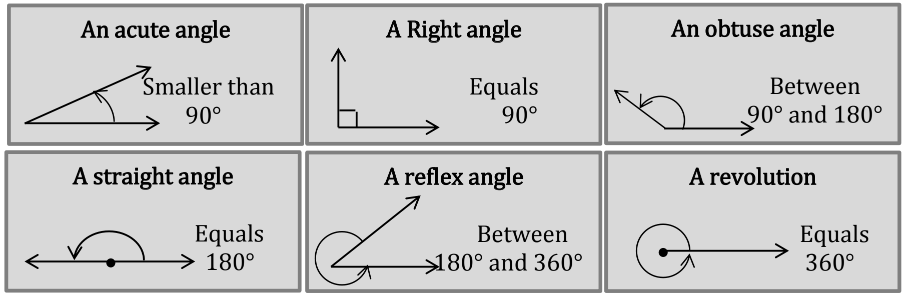
(Solutions p. 276 - 277)

CONSTRUCTIONS

1. ANGLES

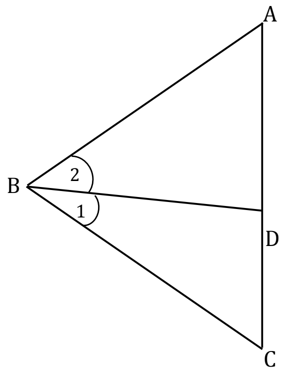
1.1 Classification of angles

Angles are classified according to their sizes.



1.2 Naming angles

We can name an angle by using the three letters on the line segments that create it. The vertex is always in the middle.



In the sketch alongside we name the top angle at B as $A\hat{B}D$ or $\hat{B}_2$.

We name the angle formed by AB and BC as $A\hat{B}C$.

We can name the angle at A as $B\hat{A}C$ but because only one angle is formed by AB and AC we can also name it by just using the vertex as $\hat{A}$.

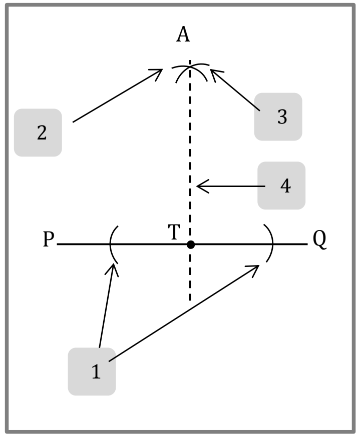
In the same way, the angle at C can be named as $\hat{C}$.

2. CONSTRUCT PERPENDICULAR LINES

2.1 Construct a perpendicular line at a given point

(Construct a perpendicular line through T on PQ)

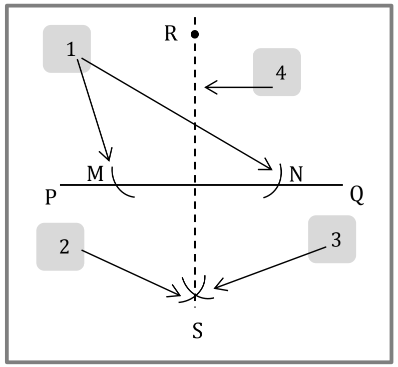
1. Place the compass on point T and draw equal arcs on opposite sides of T on PQ.
2. Place the compass on the arc to the left of T and draw an arc above the line.
3. Place the compass on the arc to the right of T and make an arc above the line so that it intersects the first arc at A.
4. Draw a line through points A and T. AP is a perpendicular line on PQ at point T.



2.2 Construct a perpendicular line from a given point

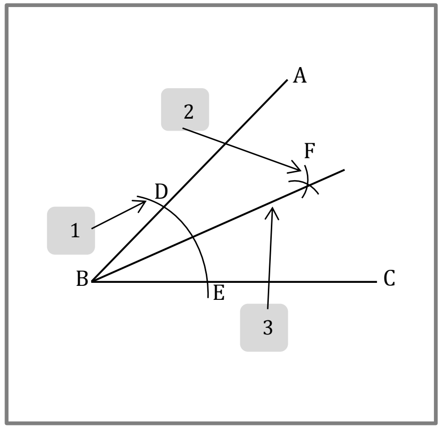
(Construct a perpendicular line from R on PQ)

1. Place your compass on point R and draw equal arcs on PQ at M and N.
2. Place your compass on M and draw an arc below line PQ.
3. Without changing the compass setting, place your compass on N and draw a second arc to intersect the first arc in S.
4. Draw a line through R and S. RS is a perpendicular line from point R on PQ.



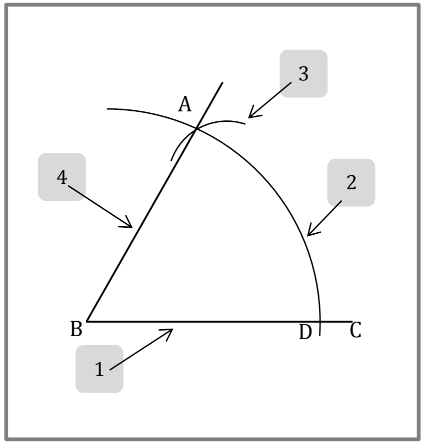
3. CONSTRUCT ANGLES

3.1 Bisect an angle



1. Place the compass on B and draw an arc that intersects AB at D and BC at E.
2. Place the compass on D and E and draw arcs of equal lengths to intersect at F.
3. Join B and F.
   BF is the bisector of $A\hat{B}C$.

3.2 Construct an angle of 60°

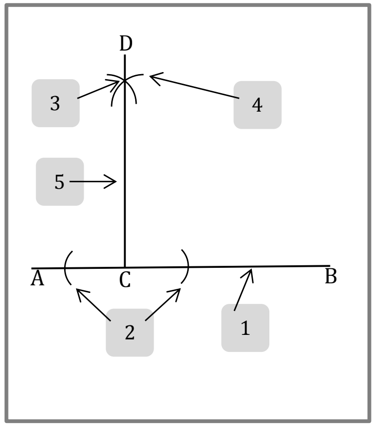


1. Draw line BC .
2. Place your compass on point B and draw a long arc that intersects BC at D.
3. Without changing your compass, place the end of your compass on D and draw a second arc that intersects the first arc at A.
4. Join AB.
   $A\hat{B}D = 60°$

3.3 Construct an angle of 30°

1. Construct an angle of 60° as in 3.2.
2. Bisect the angle of 60° as in 3.1.
3. You now have two angles of 30° each.

3.4 Construct an angle of 90°



1. Draw line AB and mark off point C between A and B.
2. Place your compass on point C and draw equal arcs on opposite sides of point C.
3. Place your compass on the arc between A and C and draw an arc above AB.
4. Without changing your compass, place the point on the arc between C and B and draw a second arc to intersect the first arc at D.
5. Join DC.
   $A\hat{C}D = B\hat{C}D = 90°$

Construct an angle of 45°

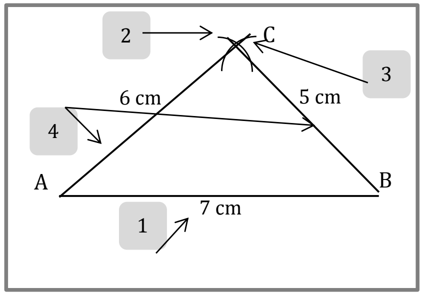
1. Construct an angle of 90° as in 3.4.
2. Bisect the angle of 90° as in 3.1.
3. You now have two angles of 45° each.

4. CONSTRUCT TRIANGLES

4.1 The length of all three sides are given

Construct ΔABC *with* $AB = 7$ cm, $BC = 5$ cm *and* $AC = 6$ cm.

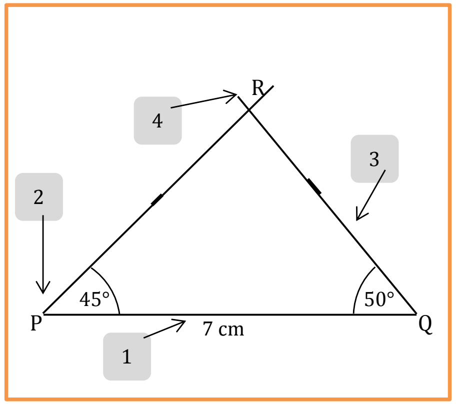
1. Draw line $AB = 7$ cm.
2. Set your compass to 6 cm, place it on A and draw an arc.
3. Set your compass to 5 cm, place it on B and draw an arc to intersect the first arc in C.
4. Join AC and BC.



4.2 Two angles and the length of one side are given

Construct ΔPQR *with* $\hat{P} = 45°, \hat{Q} = 50°$ *and* $PQ = 7$ cm

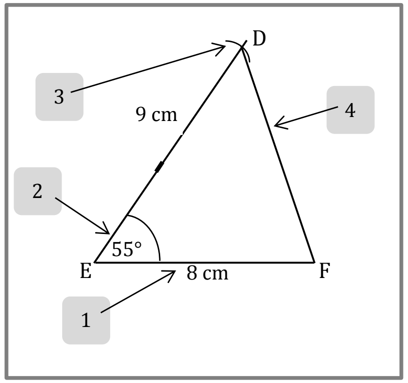
1. Draw $PQ = 7$cm.
2. At point P, measure an angle of 45°, using a protractor, and draw a line from P through the point where the angle has been marked.
3. At point Q, measure an angle of 50° and draw a line from Q through the point where the angle has been marked.
4. The point where these lines intersect is point R.
   The size of $\hat{R} = 85°$.



4.3 Two sides and one angle are given

Construct ΔDEF *with* $DE = 9$ cm, $EF = 8$ cm *and* $\hat{E} = 55°$.

1. Draw EF = 8 cm.
2. At point E, measure an angle of 55°, using a protractor, and draw a line from E through the point where the angle has been marked.
3. Set your compass to 9 cm, place it on E and draw an arc to intersect the line forming an angle of 55° with EF at D.
4. Join DF.

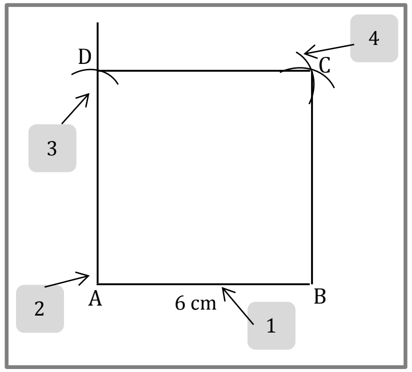


5. CONSTRUCT QUADRILATERALS

5.1 Construct a square

Construct square ABCD with sides 6 cm.

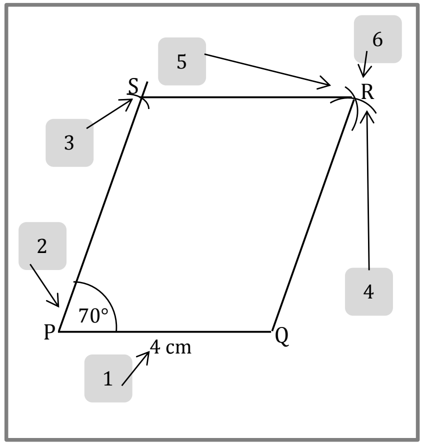
1. Draw $AB = 6$ cm.
2. At point A, construct an angle of 90° as in 3.4.
3. Set your compass to 6 cm, place the end on vertex A and draw an arc to intersects the perpendicular line at D.
4. Without changing the setting on your compass, draw arcs from B and D to intersect at C.
   Join BC and DC.



5.2 Construct a parallelogram

Construct parallelogram PQRS with $\hat{P} = 70°$, $PQ = 4$ cm *and* $QR = 6$ cm.

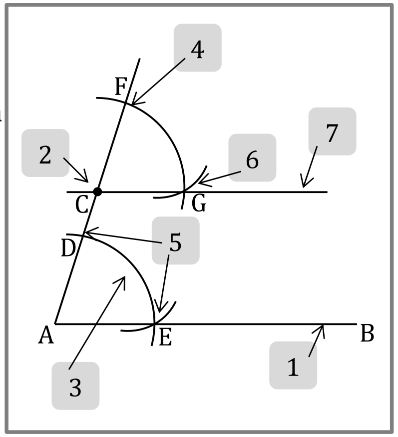
1. Draw PQ = 4 cm.
2. At P, measure an angle of 70° and draw a line from P through your mark.
3. Set your compass to 6 cm, place it on P and draw an arc that intersects the line drawn in 2 in S.
4. Place your compass on Q and draw an arc 6 cm above Q.
5. Set your compass to 4 cm, place the end on S and draw an arc to intersect the first arc in R
6. Join RS and RQ.



Practise the constructions until you are comfortable doing so. You may use other angle sizes and other lengths for the sides as well. Just make sure you are able to do the constructions.

6. CONSTRUCT PARALLEL LINES

1. Draw line AB any length.
2. Mark point C any distance from AB and draw a line from A through C.
3. Place your compass on A and draw an arc to intersects AC in D and AB in E.
4. From C, without changing the setting of your compass, draw a second arc to intersects the line through A and C in F (AD = CF).
5. Place your compass on D, make the radius equal to the length of DE and draw an arc.
6. Without changing the setting of your compass, draw an arc from F to intersects the arc through F in G.
7. Draw a line through C and G.
   $CG \parallel AB$



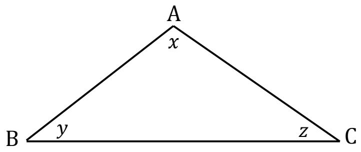
GEOMETRY OF 2D SHAPES

1. TRIANGLES

A triangle is a closed 2D(two dimensional) form with three sides and three angles. The sides can be equal in length or differ in length and the angles can be equal in size or differ in size.

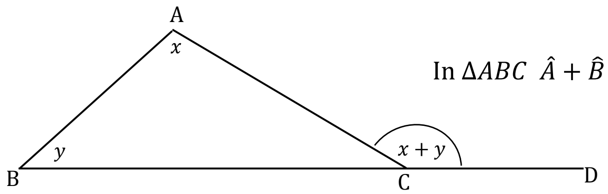
1.1. PROPERTIES OF TRIANGLES

1.1.1 **The sum of the interior angles of a triangle is equal to 180°**



In $\Delta ABC\ \hat{A} + \hat{B} + \hat{C} = 180°$;
$therefore, x + y + z = 180°$

1.1.2 **The exterior angle of a triangle is equal to the sum of the opposite interior angles**

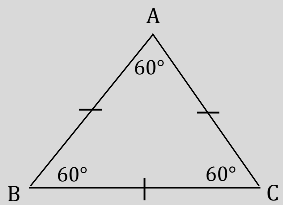


In $\Delta ABC\ \ \hat{A} + \hat{B} = A\hat{C}D$

1.1.3 **Different kinds of triangles and their properties**

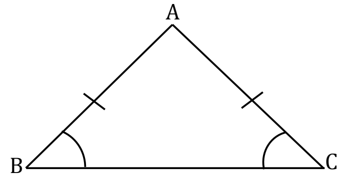
Equilateral Triangle

* All three sides are equal.
* All three angles are equal.
* All the interior angles equal 60°.



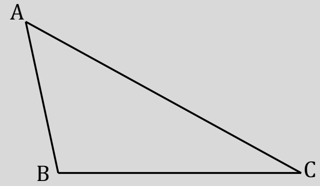
Isosceles triangle

* Two sides are equal.
* The angles opposite the equal sides are equal.



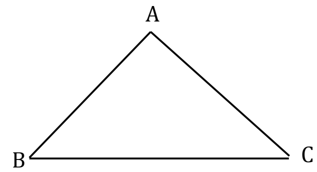
Scalene triangle

* No sides are equal.
* No angles are equal.



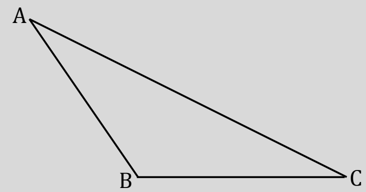
Acute-angled triangle

* All three interior angles are smaller than 90° (acute).



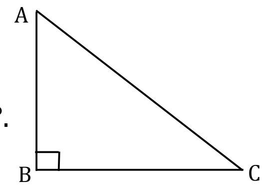
Obtuse-angled triangle

* The largest interior angle is greater than 90° (obtuse).



Right-angled triangle

* The largest interior angle equals 90°.
* The side opposite the right angle is called the hypotenuse.

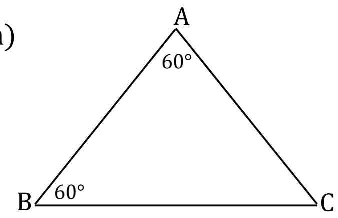


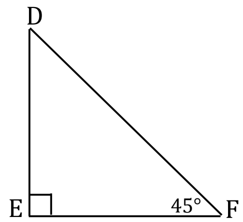
1.2 CLASSIFICATION OF TRIANGLES

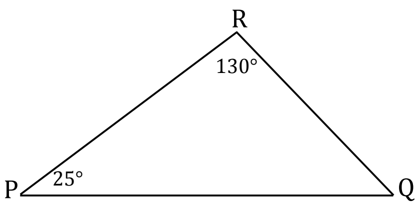
Triangles can be classified according to their angles and their sides. For example, a triangle with two equal sides and a right angle can be classified as an isosceles, right-angled triangle. Therefore, use all the information given to classify a triangle.

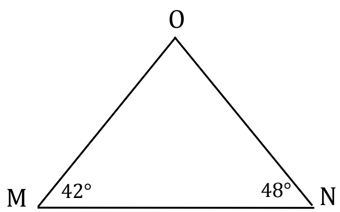
Example 1

Classify each of the following triangles:

a)


b)


c)


d)


Solution

a) In $\triangle ABC$ $\hat{C} = 180° - 60° - 60°$ $|$*Angles of a triangle* $= 180°$
$\hat{C} = 180° - 120°$
$\hat{C} = 60°$

$\triangle ABC$ is an equilateral triangle $|$*All three angles are equal*

b) In $\triangle DEF$ $\hat{D} = 180° - 90° - 45°$ $|$*Sum of the angles of* $\triangle = 180°$
$\hat{D} = 45°$
therefore $\hat{D} = \hat{F}$

$\triangle DEF$ is an isosceles, right-angled triangle $|$*Two angles are equal and one angle is a right angle* (90°)

c) In $\triangle PQR$ $\hat{Q} = 180° - 130° - 25°$ $|$*Sum of the angles of* $\triangle = 180°$
$\hat{Q} = 25°$

$\triangle PQR$ is an obtuse-angled, isosceles triangle $|$*Two equal angles and the third angle is an obtuse angle*

d) In $\triangle MNO$ $\hat{O} = 180° - 42° - 48°$ $|$*Sum of the angles of* $\triangle = 180°$
$\hat{O} = 90°$

$\triangle MNO$ is a right-angled, scalene triangle $|$*One angle* $= 90°$

1.3 DETERMINE UNKNOWN ANGLED AND SIDES OF TRIANGLES

Always remember to provide reasons for any statement you make.

You may use the following abbreviations for your reasons:

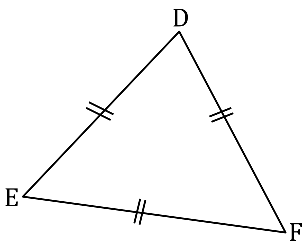
Sum of the angles of a triangle = 180°	→ ***sum of*** $\angle^s$ ***of*** Δ
Exterior angle of a triangle is equal to the sum of the opposite interior angles	→ ***exteriot***$\angle$ ***of*** Δ
Angles opposite equal sides are equal or sides opposite equal angles are equal	→ ***isosesceles*** Δ
Angles on a straight = 180°	→ $\angle^s$ ***on a straight line***
Equilateral triangle has 3 equal sides and 3 equal angles	→ ***equilateral*** Δ

You will often use algebraic equations to determine unknown angles or unknown sides. Use the information given to set up an equation and then solve the equation.

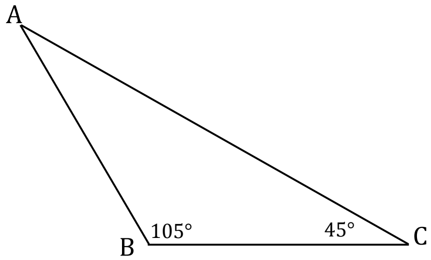
Example 2

Determine the size of the unknown angles in the following triangles:

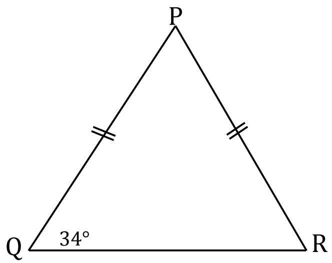
a)



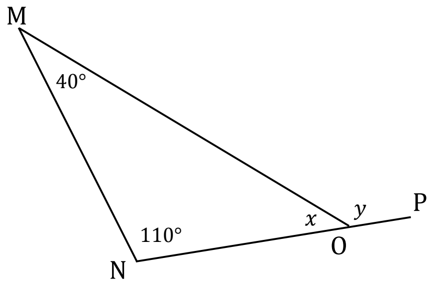
b)



c)



d)



Solution

a) $DE = DF = EF$ |given

$\underline{\hat{D} = \hat{E} = \hat{F} = 60°}$ |equilateral Δ

b) $\hat{A} + \hat{B} + \hat{C} = 180°$ |interior$\angle^s$ of a Δ = 180°

$\hat{A} + 105° + 45° = 180°$

$\hat{A} + 150° = 180°$

$\hat{A} = 180° - 150°$

$\underline{\hat{A} = 30°}$

Now solve the equation

c) $PQ = PR$ |given

$\hat{Q} = \hat{R}$ |isosceles Δ

$\underline{\hat{R} = 34°}$ |$\hat{Q} = 34°$, given

$\hat{P} + \hat{Q} + \hat{R} = 180°$ |interior$\angle^s$ of Δ= 180°

$\hat{P} + 34° + 34° = 180°$ |$\hat{Q} = 34°$, given and $\hat{R} = 34°$, proven

$\hat{P} + 68° = 180°$

$\hat{P} = 180° - 68°$

$\underline{\hat{P} = 112°}$

Now solve the equation

d) $x + 110° + 40° = 180°$ |interior$\angle^s$ of Δ= 180°

$x + 150° = 180°$

$x = 180° - 150°$

$\underline{x = 30°}$

$y + x = 180°$ |Straight angle

$y + 30° = 180°$

$y = 180° - 30°$

$\underline{y = 150°}$

2 QUADRILATERALS

A quadrilateral is a closed-plane figure bounded by four sides.

The sum of the interior angles of any quadrilateral is equal to 360°.

2.1 PROPERTIES OF QUADRILATERALS

1. **A trapezium** is a quadrilateral of which
   * one pair of opposite sides are parallel but not equal.